Chapter 2: Data

What are data?

In order to determine the context of data, consider the "W's"

- Who –
- What (and in what units) –
- When –
- Where –
- Why –
- How –

There are two major ways to treat data:

- A _____ _____ is used to answer questions about how cases fall into categories. A categorical variable may be comprised of word labels, or it may use numbers as labels.
  Examples:
- A _____ _____ is used to answer questions about the quantity of what is being measured. A quantitative variable is comprised of numeric values.
  Examples:

What is a statistic?

Are the numbers **17, 21, 44, 76** data?

Data must have _____ to be meaningful. The numbers listed above could be test scores, ages of a group of golfers, or the uniform numbers of the starting backfield on the football team. Without _____ data cannot be interpreted.

Suppose a Consumer Reports article (published in June 2005) on energy bars gave the brand name, flavor, price, number of calories, and grams of protein and fat. Identify the following:

- Who:
- What:
- When:
- Where:
- How:
- Why:
- Categorical variables:
- Quantitative variables (with units):

A report on the Boston Marathon listed each runner's gender, county, age, and time. Identify the following:

- Who:
- What:
- When:
- Where:
- How:
- Why:
- Categorical variables:
- Quantitative variables (with units):

Chapter 2: Data

What are data?
Data are values along with their context. Data can be numbers or labels.

In order to determine the context of data, consider the "W's"

- Who – *the cases (about whom the data was collected). People are referred to as* ***respondents****,* ***subjects,*** *or* ***participants****, while objects are referred to as* ***experimental units.***
- What (and in what units) – *the variables recorded about each individual.*
- When – *when the data was collected.*
- Where – *where the data was collected.*
- Why – *why the data was collected. This can determine whether a variable is treated as* ***categorical*** *or* ***quantitative****.*
- How – *how the data was collected.*

There are two major ways to treat data: ***categorical*** and ***quantitative***.

- A *categorical variable* names categories and is used to answer questions about how cases fall into those categories. A categorical variable may be comprised of word labels, or it may use numbers as labels.
- A *quantitative variable* is used to answer questions about the quantity of what is being measured. A quantitative variable is comprised of numeric values.

What is a statistic? *A statistic is a numerical summary of data.*

17, 21, 44, 76

Are the numbers listed above data? Data must have *context* to be meaningful. The numbers listed above could be test scores, ages of a group of golfers, or the uniform numbers of the starting backfield on the football team. Without *context*, data cannot be interpreted.

Suppose a Consumer Reports article (published in June 2005) on energy bars gave the brand name, flavor, price, number of calories, and grams of protein and fat. Identify the following:

- Who: *energy bars*
- What: *brand, flavor, price, calories, protein, fat*
- When: *not specified*
- Where: *not specified*
- How: *not specified (nutrition label? laboratory testing?)*
- Why: *to inform potential consumers*
- Categorical variables: *brand, flavor*

- Quantitative variables (with units): *price (US$), number of calories (calories), protein (grams), fat (grams)*

A report on the Boston Marathon listed each runner's gender, county, age, and time. Identify the following:

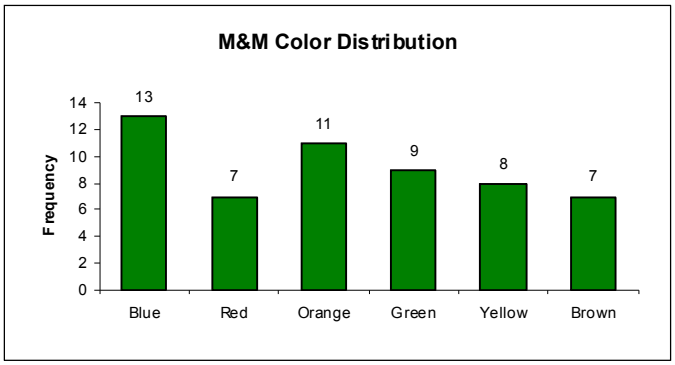
- Who: *Boston Marathon runners*
- What: *gender, county, age, time*
- When: *not specified*
- Where: *Boston*
- How: *not specified (registration information?)*
- Why: *race result reporting*
- Categorical variables: *gender, county*
- Quantitative variables (with units): *age (years), time (hours, minutes, seconds)*

Chapter 3 – Displaying and Describing Categorical Data

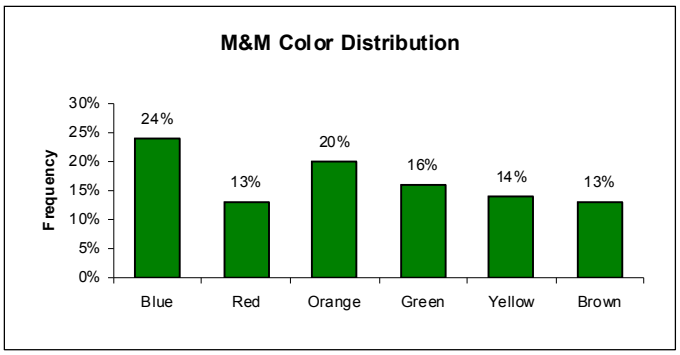
_____ _____ are often used to organize categorical data. Frequency tables display the category names and the _____ of the number of data values in each category. _____ also display the category names, but they give the _____ rather than the counts for each category.

Color	Freq.	Rel. Freq.	Percent
Blue	13		
Red	7		
Orange	11		
Green	9		
Yellow	8		
Brown	7		
TOTAL	**55**	**1.000**	**100%**

A _____ is often used to display categorical data. The height of each bar represents the _____ for each category. Bars are displayed next to each other for easy comparison. When constructing a bar chart, note that the bars do not _____ one another. Categorical variables usually cannot be ordered in a meaningful way; therefore the order in which the bars are displayed is often meaningless.



A _____ bar chart displays the proportion of counts for each category.



The sum of the relative frequencies is _____.

A _____ _____ is another type of display used to show categorical data. Pie charts show parts of a whole. Pie charts are often difficult to construct by hand.

A _____ _____ shows two categorical variables together. The margins give the frequency distributions for each of the variables, also called the _____ .

Examine the class data about gender and political view – liberal, moderate, conservative.

	Liberal	**Moderate**	**Conservative**	**TOTAL**
Male				
Female				
TOTAL				

- What percent of the class are girls with liberal political views?
- What percent of the liberals are girls?
- What percent of the girls are liberals?
- What is the marginal distribution of gender?
- What is the marginal distribution of political views?

A conditional distribution shows the distribution of one variable for only the individuals who satisfy some condition on another variable.

The conditional distribution of political preference, conditional on being male:

	Liberal	**Moderate**	**Conservative**	**TOTAL**
Male				

The conditional distribution of political preference, conditional on being female:

	Liberal	**Moderate**	**Conservative**	**TOTAL**
Female				

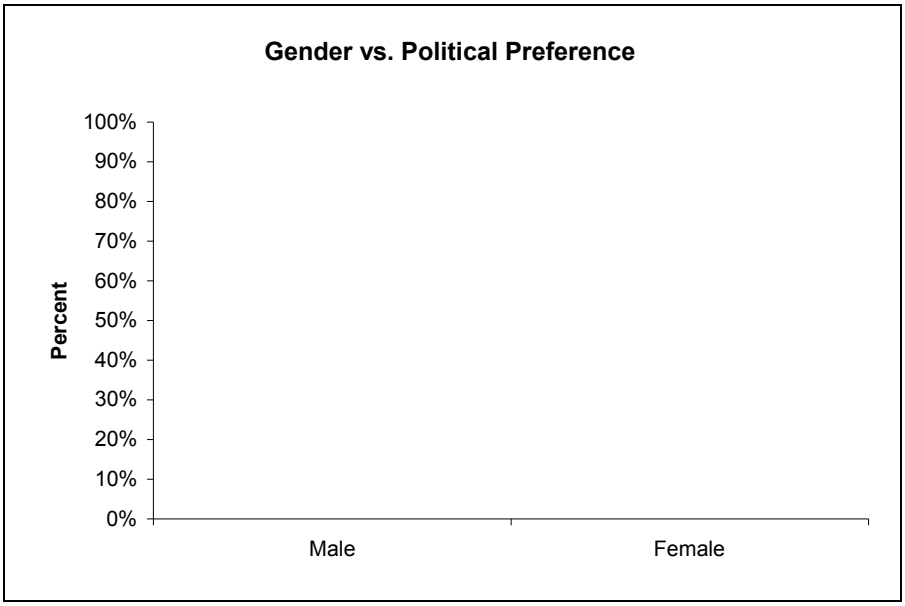
- What is the conditional relative frequency distribution of gender among conservatives?

If the conditional distributions are the same, we can conclude that the variables are not associated. Therefore, they are _____ of one another.

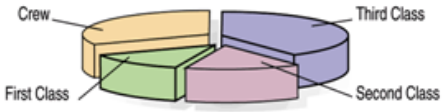
If the conditional distributions differ, we can conclude that the variables are somehow associated. Therefore, they are _____ of one another.

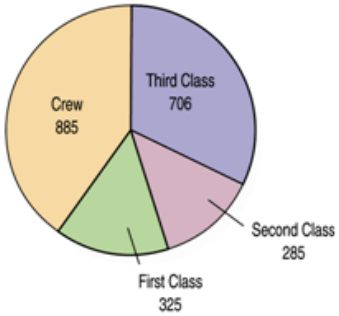
- Are gender and political view independent?

A segmented bar chart displays the same information as a pie chart, but in the form of bars instead of circles. Comparing segmented bar charts is a good way to tell if two variables are independent of one another or not.

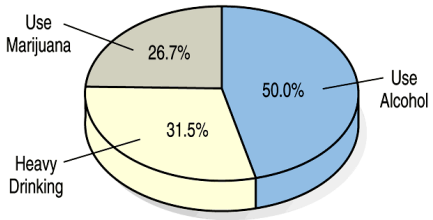


- Explain how the graph on the left violates the "area principle."





- Explain what is wrong with the graph below.



Averaging one variable across different levels of a second variable can lead to _____. Consider the following example:

It's the last inning of an important game. Your team is a run down with the bases loaded and two outs. The pitcher is due up, so you'll be sending in a pinch-hitter. There are 2 batters available on the bench. Whom should you send in to bat?

Player	Overall
A	33 for 103
B	45 for 151

- Compare A's batting average to B's batting average. Which player appears to be the better choice?

Does it matter whether the pitcher throws right- or left-handed?

Player	Overall	vs LHP	vs RHP
A	33 for 103	28 for 81	5 for 22
B	45 for 151	12 for 32	33 for 119

- Compare A's batting average vs. a left-handed pitcher to B's. Compare A's batting average against a right-handed pitcher. Which player appears to be the better choice?

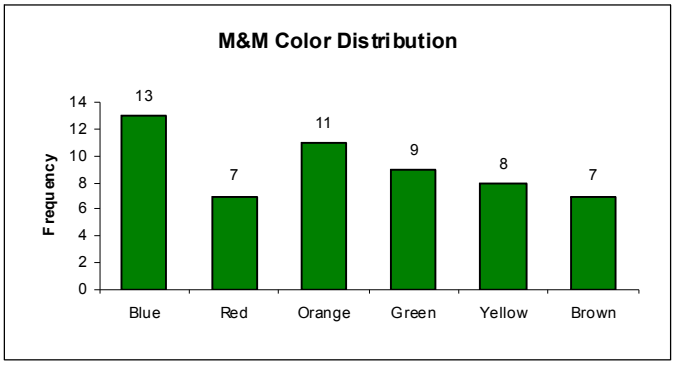
Pooling the data together loses important information and sometimes leads to the wrong conclusion. We always should take into account any factor that might matter.

Notes: Displaying and Describing Categorical Data

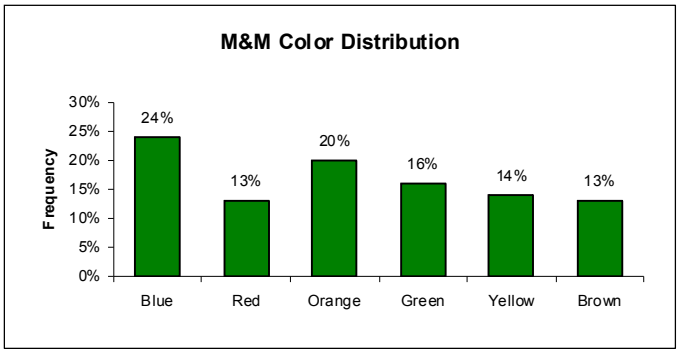
Frequency tables are often used to organize categorical data. Frequency tables display the category names and the counts of the number of data values in each category. Relative frequency tables also display the category names, but they give the percentages rather than the counts for each category.

Color	Freq.	Rel. Freq.	Percent
Blue	13	0.236	24%
Red	7	0.127	13%
Orange	11	0.200	20%
Green	9	0.164	16%
Yellow	8	0.145	14%
Brown	7	0.127	13%
TOTAL	**55**	**1.000**	**100%**

A bar chart is often used to display categorical data. The height of each bar represents the count for each category. Bars are displayed next to each other for easy comparison. When constructing a bar chart, note that the bars do not touch one another. Categorical variables usually cannot be ordered in a meaningful way; therefore the order in which the bars are displayed is often meaningless.



A relative frequency bar chart displays the proportion of counts for each category.



The sum of the relative frequencies is 100%.

A pie chart is another type of display used to show categorical data. Pie charts show parts of a whole. Pie charts are often difficult to construct by hand.

A contingency table shows two categorical variables together. The margins give the frequency distributions for each of the variables, also called the marginal distribution.

Examine the class data about gender and political view – liberal, moderate, conservative.

	Liberal	**Moderate**	**Conservative**	**TOTAL**
Male				
Female				
TOTAL				

- What percent of the class are girls with liberal political views?
- What percent of the liberals are girls?
- What percent of the girls are liberals?
- What is the marginal distribution of gender?
- What is the marginal distribution of political views?

A conditional distribution shows the distribution of one variable for only the individuals who satisfy some condition on another variable.

The conditional distribution of political preference, conditional on being male:

	Liberal	**Moderate**	**Conservative**	**TOTAL**
Male				

The conditional distribution of political preference, conditional on being female:

	Liberal	**Moderate**	**Conservative**	**TOTAL**
Female				

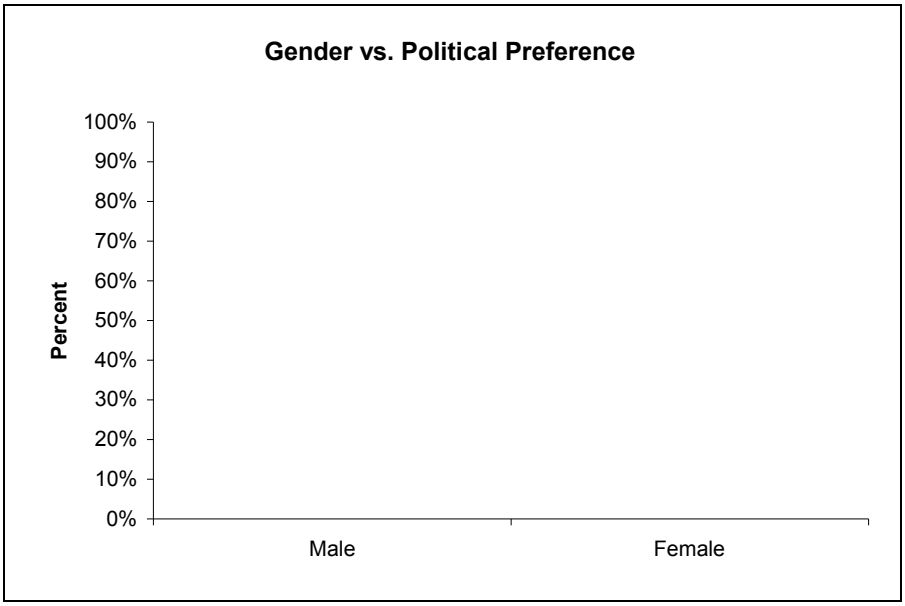
- What is the conditional relative frequency distribution of gender among conservatives?

If the conditional distributions are the same, we can conclude that the variables are not associated. Therefore, they are independent of one another.

If the conditional distributions differ, we can conclude that the variables are somehow associated. Therefore, they are not independent of one another.

- Are gender and political view independent?

A segmented bar chart displays the same information as a pie chart, but in the form of bars instead of circles. Comparing segmented bar charts is a good way to tell if two variables are independent of one another or not.



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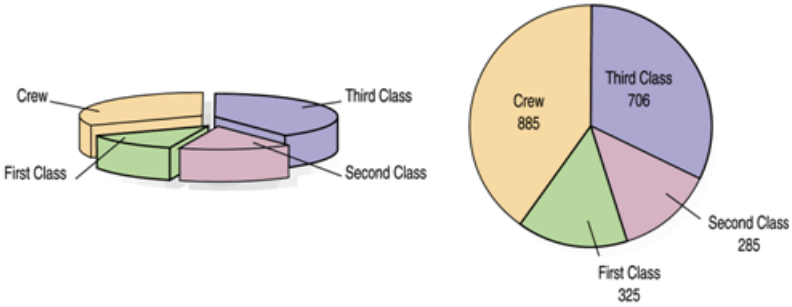

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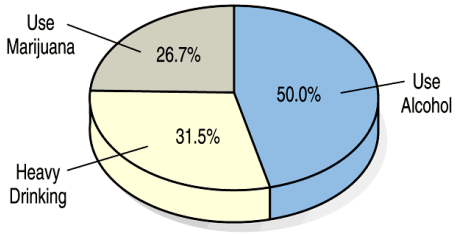


Averaging one variable across different levels of a second variable can lead to **Simpson's Paradox**. Consider the following example:

It's the last inning of an important game. Your team is a run down with the bases loaded and two outs. The pitcher is due up, so you'll be sending in a pinch-hitter. There are 2 batters available on the bench. Whom should you send in to bat?

Player	Overall
A	33 for 103
B	45 for 151

- Compare A's batting average to B's batting average. Which player appears to be the better choice?
  Player A has a higher batting average (0.320 vs. 0.298), so he looks like the better choice.

Does it matter whether the pitcher throws right- or left-handed?

Player	Overall	vs LHP	vs RHP
A	33 for 103	28 for 81	5 for 22
B	45 for 151	12 for 32	33 for 119

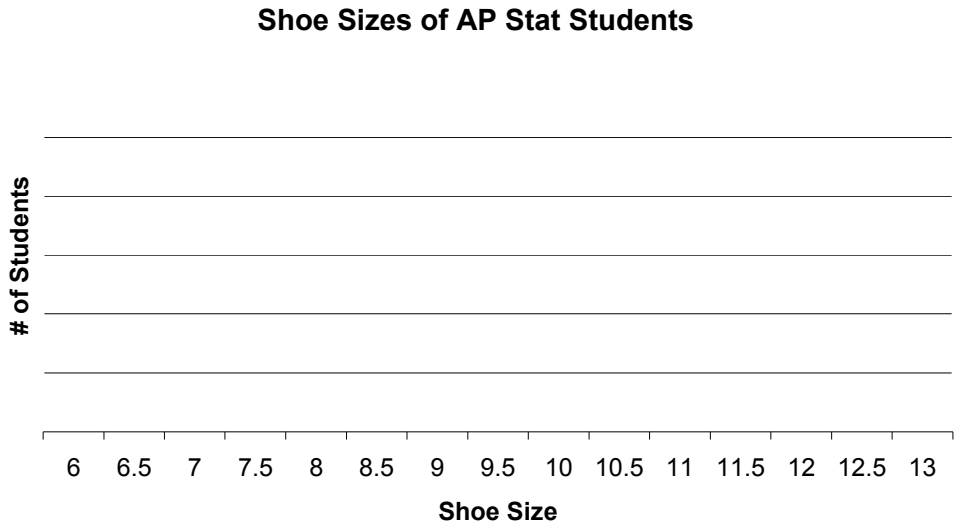
- Compare A's batting average vs. a left-handed pitcher to B's. Compare A's batting average against a right-handed pitcher. Which player appears to be the better choice?
  Player B has a higher batting average against both right- and left-handed pitching, even though his overall average is lower. Player B hits better against both right- and left-handed pitchers. So no matter the pitcher, B is a better choice. So why is his batting "average" lower? Because B sees a lot more right-handed pitchers than A, and (at least for these guys) right-handed pitchers are harder to hit. For some reason, A is used mostly against left-handed pitchers, so A has a higher average.

Pooling the data together loses important information and leads to the wrong conclusion. We always should take into account any factor that might matter.

Notes: Displaying Quantitative Data

A _____ or _____ is often used to display categorical data. These types of displays, however, are not appropriate for quantitative data. Quantitative data is often displayed using either a _____ _____ or a _____

In a histogram, the interval corresponding to the width of each bar is called a _____ A histogram displays the bin counts as the height of the bars (like a bar chart). Unlike a bar chart, however, the bars in a histogram _____ one another. An empty space between bars represents a _____ in data values. If a value falls on the border between two consecutive bars, it is placed in the bin on the _____.



A _____ _____ histogram displays the proportion of cases in each bin instead of the count.

Histograms are useful when _____, and they can easily be constructed using a graphing calculator. A disadvantage of histograms is that they _____ _____.

Be sure to choose an appropriate bin width when constructing a histogram. As a general rule of thumb, your histogram should contain about _____ bars.

A _____ _____ is similar to a histogram, but it shows_____ _____ rather than bars. It may be necessary to _____ stems if the range of data values is small.

Number of Pairs of Shoes Owned

0
0
1
1
2
2
3
3

KEY:

A _____ _____ stem-and-leaf plot can be useful when _____ two distributions.

Number of Pairs of Shoes Owned

Male	Female
	0
	0
	1
	1
	2
	2
	3
	3

KEY:

The stems of the stem-and-leaf plot correspond to the _____ of a histogram. You may only use _____ digit for the leaves. Round or truncate your values if necessary.

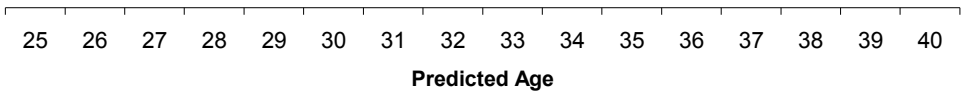
Stem-and-leaf plots are useful when working with sets of data that are _____ _____ in size, and when you want to display _____ _____.

How would you setup the following stem-and-leaf plots?

- quiz scores (out of 100)
- student GPA's
- student weights
- SAT scores
- weights of cattle (1000-2000 pounds)

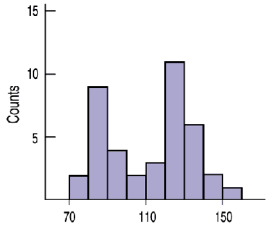
_____ may also be used to display quantitative variables. Dot plots are useful when working with _____ sets of data.

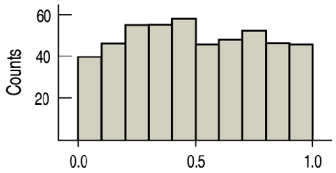
Guess Your Teacher's Age

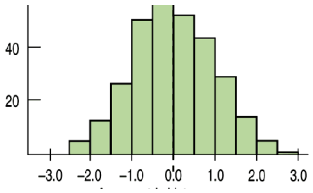


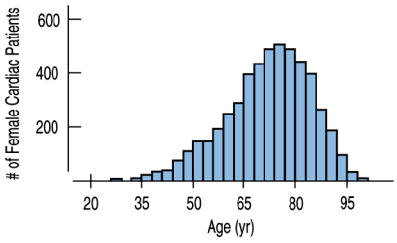
When describing a distribution, you should tell about three things: _____ , _____, and _____. You should also mention any unusual features, like _____ or _____.

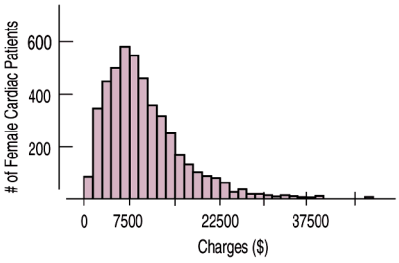
Identify the shapes of the following distributions:











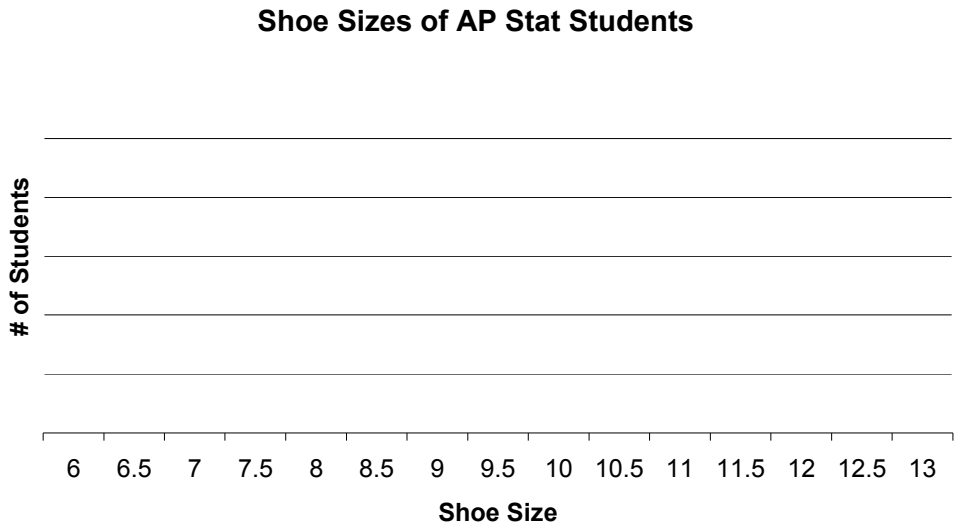
When comparing two or more distributions, compare the _____ , _____, and _____, and compare any _____ features. It is important, when comparing distributions, that their graphs be constructed using the same _____.

You can sometimes make a skewed distribution appear more symmetric by _____ (or transforming) your data.

Notes: Displaying Quantitative Data

A bar chart or pie chart is often used to display categorical data. These types of displays, however, are not appropriate for quantitative data. Quantitative data is often displayed using either a histogram, dot plot, or a stem-and-leaf plot.

In a histogram, the interval corresponding to the width of each bar is called a bin. A histogram displays the bin counts as the height of the bars (like a bar chart). Unlike a bar chart, however, the bars in a histogram touch one another. An empty space between bars represents a gap in data values. If a value falls on the border between two consecutive bars, it is placed in the bin on the right.



A relative frequency histogram displays the proportion of cases in each bin instead of the count.

Histograms are useful when working with large sets of data, and they can easily be constructed using a graphing calculator. A disadvantage of histograms is that they do not show individual values.

Be sure to choose an appropriate bin width when constructing a histogram. As a general rule of thumb, your histogram should contain about 10 bars.

A stem-and-leaf plot is similar to a histogram, but it shows individual values rather than bars. It may be necessary to split stems if the range of data values is small.

Number of Pairs of Shoes Owned

0
0
1
1
2
2
3
3

KEY:

A back-to-back stem-and-leaf plot can be useful when comparing two distributions.

Number of Pairs of Shoes Owned

Male	Female
	0
	0
	1
	1
	2
	2
	3
	3

KEY:

The stems of the stem-and-leaf plot correspond to the bins of a histogram. You may only use one digit for the leaves. Round or truncate your values if necessary.

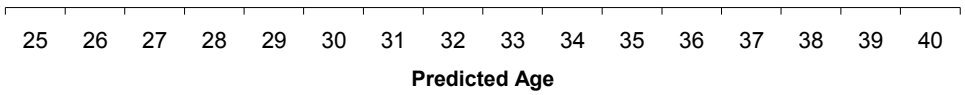
Stem-and-leaf plots are useful when working with sets of data that are small to moderate in size, and when you want to display individual values.

How would you setup the following stem-and-leaf plots?

- quiz scores (out of 100)
- student GPA's
- student weights
- SAT scores
- weights of cattle (1000-2000 pounds)

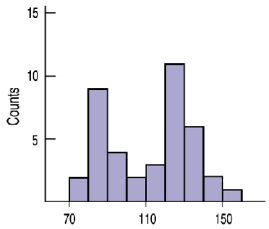
Dot plots may also be used to display quantitative variables. Dot plots are useful when working with small sets of data.

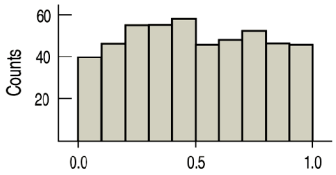
Guess Your Teacher's Age

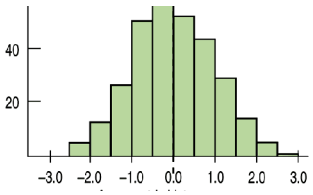


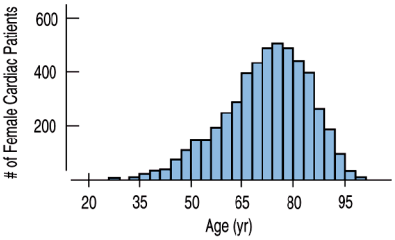
When describing a distribution, you should tell about three things: shape, center, and spread. You should also mention any unusual features, like outliers or gaps.

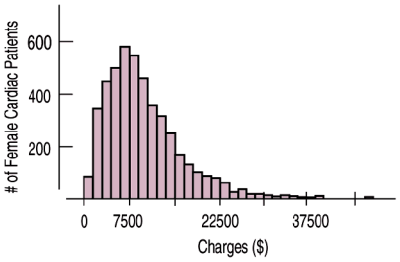
Identify the shapes of the following distributions:











When comparing two or more distributions, compare the shapes, centers, and spreads, and compare any unusual features. It is important, when comparing distributions, that their graphs be constructed using the same scale.

You can sometimes make a skewed distribution appear more symmetric by re-expressing (or transforming) your data.

Notes: Describing Distributions Numerically

When describing distributions, we need to discuss _____, _____, and _____. How we measure the center and spread of a distribution depends on its _____. The center of a distribution is a "typical" value. If the shape is unimodal and symmetric, a "typical" value is in the _____. If the shape is skewed, however, a "typical" value is not necessarily in the middle.

For _____ distributions, use the _____ to determine the _____ of the distribution and the _____ to describe the _____ of the distribution.

The median:

- is the _____ data value (when the data have been _____) that divides the histogram into two equal _____
- has the same _____ as the data
- is _____ to outliers (extreme data values)

The range:

- is the difference between the _____ value and the _____ value
- is a _____, NOT an _____
- is _____to outliers

The interquartile range (IQR):

- contains the _____ of the data
- is the difference between the _____ and _____ quartiles
- is a _____, NOT an _____
- is _____ to outliers

The _____ _____ gives: _____, _____, _____, _____, _____,

A graphical display of the five-number summary is called a _____.

How many hours, on average, do you spend watching TV per week? _____ Collect data from the entire class and record the values in order from smallest to largest. Calculate the five-number summary:

Construct both a histogram and a boxplot (using the same scale). Compare the displays.

Average Number of Hours per Week Spent Watching TV

For _____ distributions, use the _____ _to determine the _____ of the distribution and the _____ to describe the _____ of the distribution.

The mean:

- is the arithmetic _____ of the data values
- is the _____ of a histogram
- has the same _____ as the data
- is _____ to outliers
- is given by the formula

The standard deviation:

- measures the "typical" distance each data value is from the _____
- Because some values are above the mean and some are below the mean, finding the sum is not useful (positives cancel out negatives); therefore we first _____ the deviations, then calculate an _____ _____ . This is called the _____. This statistics does not have the same units as the data, since we squared the deviations. Therefore, the final step is to take the _____ of the variance, which gives us the _____ _____.
- is given by the formula
- is _____to outliers, since its calculation involves the _____

Find the mean and standard deviation of the average number of hours spent watching TV per week for this class.

Notes: Describing Distributions Numerically

When describing distributions, we need to discuss shape, center, and spread. How we measure the center and spread of a distribution depends on its shape. The center of a distribution is a "typical" value. If the shape is unimodal and symmetric, a "typical" value is in the middle. If the shape is skewed, however, a "typical" value is not necessarily in the middle.

For skewed distributions, use the median to determine the center of the distribution and the interquartile range to describe the spread of the distribution.

The median:

- is the middle data value (when the data have been ordered) that divides the histogram into two equal areas
- has the same units as the data
- is resistant to outliers (extreme data values)

The range:

- is the difference between the maximum value and the minimum value
- is a number, NOT an interval
- is sensitive to outliers

The interquartile range (IQR):

- contains the middle 50% of the data
- is the difference between the lower (Q1) and upper (Q3) quartiles
- is a number, NOT an interval
- is resistant to outliers

The Five-Number Summary gives: minimum, lower quartile, median, upper quartile, maximum.

A graphical display of the five-number summary is called a boxplot.

How many hours, on average, do you spend watching TV per week? Collect data from the entire class and record the values in order from smallest to largest. Calculate the five-number summary:

Construct both a histogram and a boxplot (using the same scale). Compare the displays.

Average Number of Hours per Week Spent Watching TV

For symmetric distributions, use the mean to determine the center of the distribution and the standard deviation to describe the spread of the distribution.

The mean:

- is the arithmetic average of the data values
- is the balancing point of a histogram
- has the same units as the data
- is sensitive to outliers
- is given by the formula $\bar{x} = \frac{\sum x}{n}$

The standard deviation:

- measures the "typical" distance each data value is from the mean
- Because some values are above the mean and some are below the mean, finding the sum is not useful (positives cancel out negatives); therefore we first square the deviations, then calculate an adjusted average. This is called the variance. This statistics does not have the same units as the data, since we squared the deviations. Therefore, the final step is to take the square root of the variance, which gives us the standard deviation.
- is given by the formula $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$
- is sensitive to outliers, since its calculation involves the mean

Find the mean and standard deviation of the average number of hours spent watching TV per week for this class.

Notes: Standard Deviation and the Normal Model

Standard deviation is a measure of spread, or _____. The smaller the standard deviation, the _____ variability is present in the data. The larger the standard deviation, the _____ variability is present in the data.

Standard deviation can be used as a ruler for measuring how an individual compares to a _____.

To measure how far above or below the mean any given data value is, we find its _____, or _____.

$$z =$$

To standardize a value, subtract the _____and divide by the _____.

Measure your height in inches. Calculate the standardized value for your height given that the average height for women is 64.5 inches with a standard deviation of 2.5 inches and for men is 69 inches with a standard deviation of 2.5 inches. *Are you tall?*

$$z_{height} =$$

Suppose the average woman's shoe size is 8.25 with a standard deviation 1.15 and the average male shoe size is 10 with a standard deviation of 1.5. *Do you have big feet?*

$$z_{shoe} =$$

Suppose Sharon wears a size 9 shoe and Andrew wears a size 9. *Does Sharon have big feet? Does Andrew?*

$$z_{Sharon} = \qquad\qquad z_{Andrew} =$$

In order to compare values that are measured using different scales, you must first _____the values. The standardized values have no _____and are called _____. Z-scores represent how far the value is above the _____ (if _____) or below the _____(if _____).

Example: z = 1 means the value is _____standard deviation _____the mean
z = -0.5 means the value is _____of a standard deviation _____the mean

The _____the z-score, the more unusual it is.

Standardized values, because they have no units, are therefore useful when comparing values that are measured on different _____, with different _____, or from different _____.

Adding a constant to all of the values in a set of data adds the same constant to the measures of _____. It does not, however, affect the _____.

Example: Add 5 to each value in the given set of data (on the left) to form a new set of data (on the right). Then find the indicated measures of center and spread.

{5, 5, 10, 35, 45}.	**{____, ____, ____, ____, ____}.**
Center:	Center:
$\bar{x}$ =	$\bar{x}$ =
M =	M =
Mode =	Mode =
Spread:	Spread:
Range =	Range =
IQR =	IQR =
SD =	SD =

Multiplying a constant to all of the values in a set of data multiplies the same constant to the measures of _____and _____.

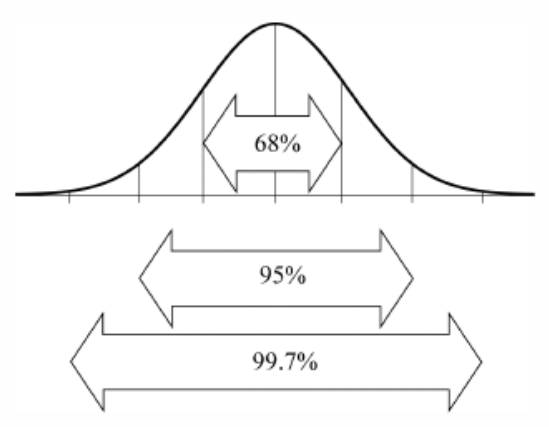
Example: Multiply each value in the given set of data (on the left) by 2 to form a new set of data (on the right). Then find the indicated measures of center and spread.

{5, 5, 10, 35, 45}.	**{____, ____, ____, ____, ____}.**
Center:	Center:
$\bar{x}$ =	$\bar{x}$ =
M =	M =
Mode =	Mode =
Spread:	Spread:
Range =	Range =
IQR =	IQR =
SD =	SD =

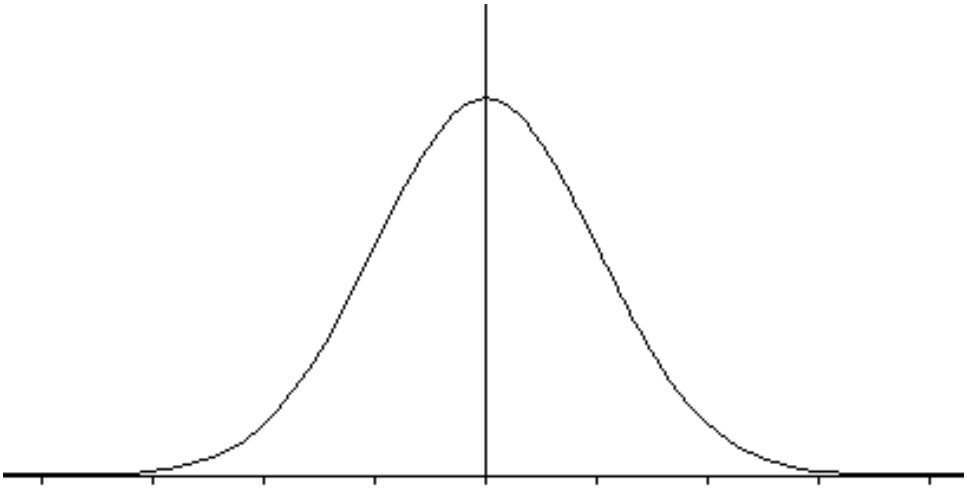
By standardizing values, we shift the distribution so that the mean is _____, and rescale it so that the standard deviation is _____. Standardizing does not change the _____of the distribution.

The Normal model:

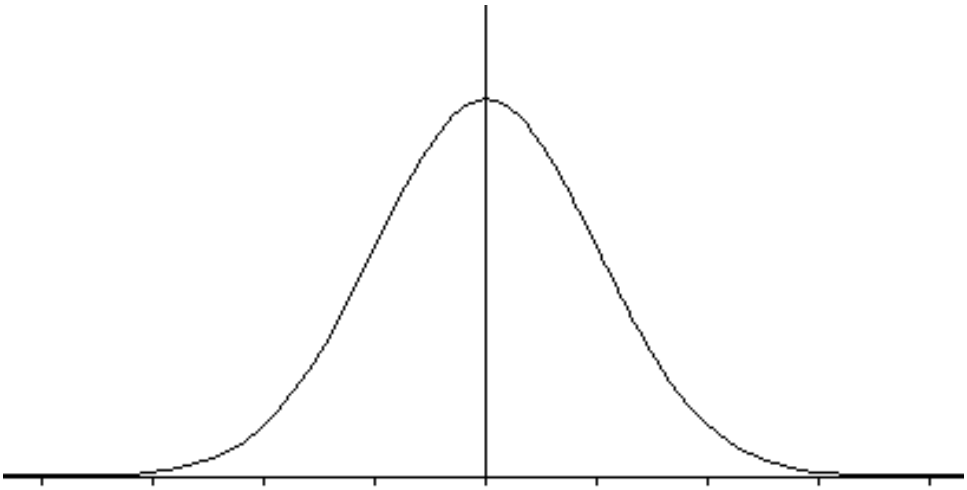
- is _____and _____.
- follows the _____
  - About _____ of the values fall within _____ standard deviation of the mean.
  - About _____ of the values fall within _____standard deviations of the mean.
  - About _____ (almost all) of the values fall within _____standard deviations of the mean.



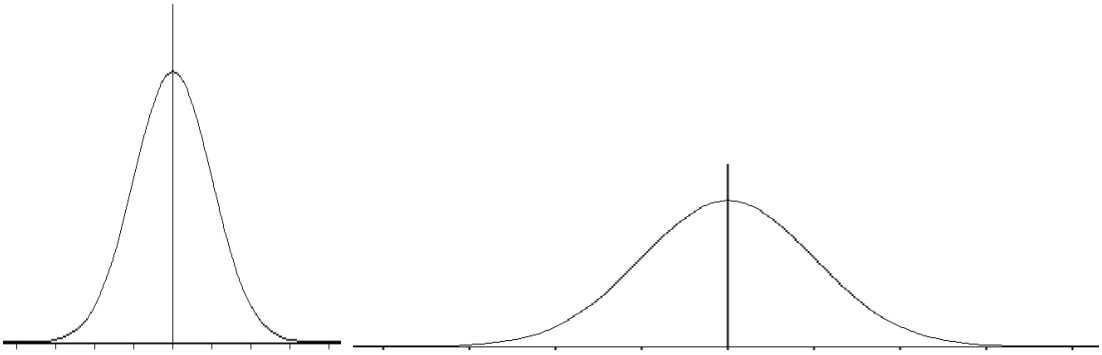
The standard Normal model has mean _____ and standard deviation _____.



The Normal model is determined by _____and _____. We use the Greek letters sigma and mu because this is a _____; it does not come from actual _____. Sigma and mu are the _____that specify the model.



The larger sigma, the _____spread out the normal model appears. The inflection points occur a distance of _____on either side of _____.



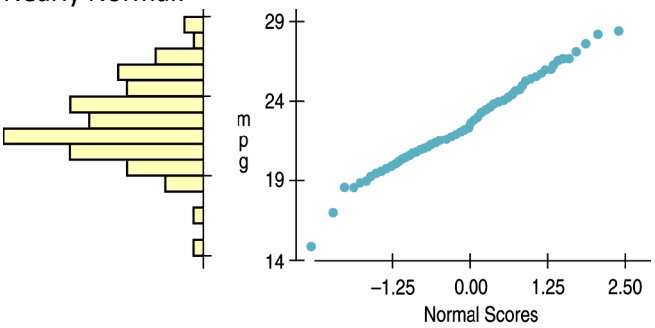
To standardize Normal data, subtract the _____ (_____) and divide by the _____ (_____).

$$z =$$

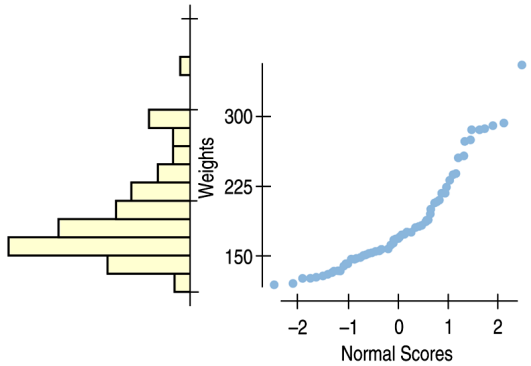
To assess normality:

- Examine the _____of the histogram or stem-and-leaf plot. A normal model is _____about the mean and _____.
- Compare the mean and median. In a Normal model, the mean and median are _____.
- Verify that the _____holds.
- Construct a _____. If the graph is linear, the model is Normal.

Nearly Normal:



Skewed distribution:



Notes: Standard Deviation and the Normal Model

Standard deviation is a measure of spread, or **variability**. The smaller the standard deviation, the **less** variability is present in the data. The larger the standard deviation, the **more** variability is present in the data.

Standard deviation can be used as a ruler for measuring how an individual compares to a **group**.

To measure how far above or below the mean any given data value is, we find its **standardized value**, or **z-score**.

$$z = \frac{y - \bar{y}}{s}$$

To standardize a value, subtract the **mean** and divide by the **standard deviation**.

Measure your height in inches. Calculate the standardized value for your height given that the average height for women is 64.5 inches with a standard deviation of 2.5 inches and for men is 69 inches with a standard deviation of 2.5 inches. *Are you tall?*

$$z_{height} =$$

Suppose the average woman's shoe size is 8.25 with a standard deviation 1.15 and the average male shoe size is 10 with a standard deviation of 1.5. *Do you have big feet?*

$$z_{shoe} =$$

Suppose Sharon wears a size 9 shoe and Andrew wears a size 9. *Does Sharon have big feet? Does Andrew?*

$$z_{Sharon} = \qquad\qquad z_{Andrew} =$$

In order to compare values that are measured using different scales, you must first **standardize** the values. The standardized values have no **units** and are called **z-scores**. Z-scores represent how far the value is above the **mean** (if **positive**) or below the mean (if **negative**).

Ex: z = 1 means the value is **one** standard deviation **above** the mean
z = -0.5 means the value is **one-half** of a standard deviation **below** the mean

The **larger** the z-score, the more unusual it is.

Standardized values, because they have no units, are therefore useful when comparing values that are measured on different **scales**, with different **units**, or from different **populations**.

Adding a constant to all of the values in a set of data adds the same constant to the measures of **center** and percentiles. It does not, however, affect the **spread**.

Example: Add 5 to each value in the given set of data (on the left) to form a new set of data (on the right). Then find the indicated measures of center and spread.

{5, 5, 10, 35, 45}.	**{10, 10, 15, 40, 50}.**
Center:	Center:
$\bar{x}$ =	$\bar{x}$ =
M =	M =
Mode =	Mode =
Spread:	Spread:
Range =	Range =
IQR =	IQR =
SD =	SD =

Multiplying a constant to all of the values in a set of data multiplies the same constant to the measures of **center** and **spread**.

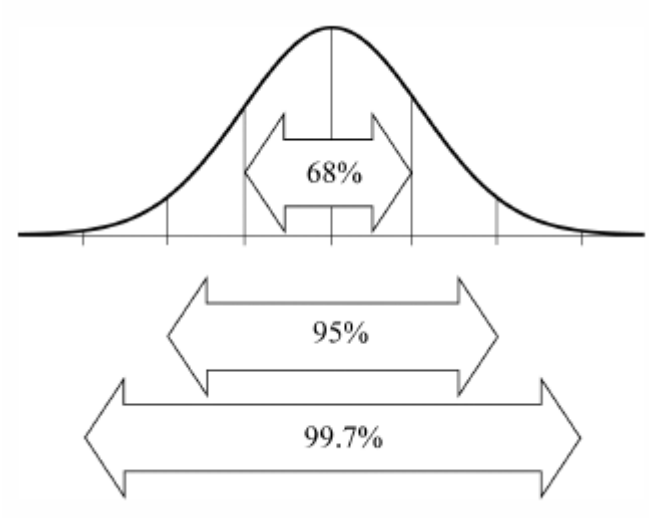
Example: Multiply each value in the given set of data (on the left) by 2 to form a new set of data (on the right). Then find the indicated measures of center and spread.

{5, 5, 10, 35, 45}.	**{10, 10, 20, 70, 90}.**
Center:	Center:
$\bar{x}$ =	$\bar{x}$ =
M =	M =
Mode =	Mode =
Spread:	Spread:
Range =	Range =
IQR =	IQR =
SD =	SD =

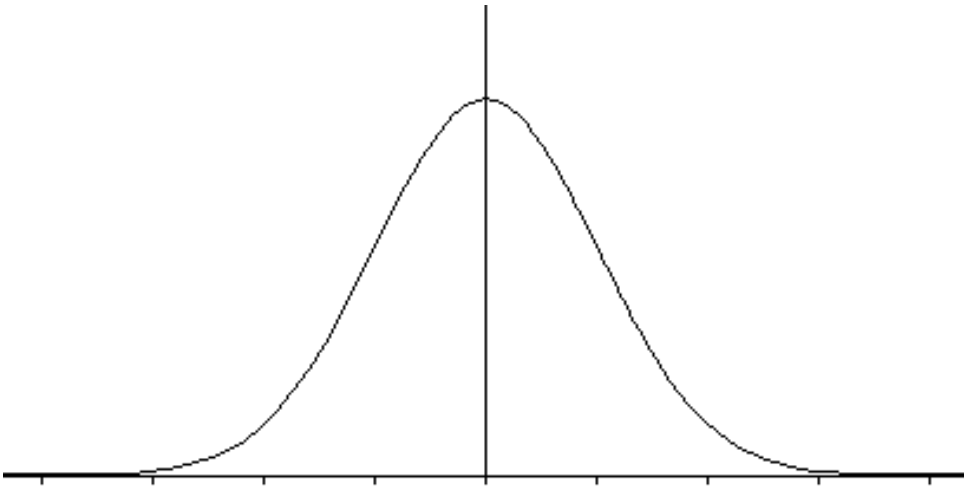
By standardizing values, we shift the distribution so that the mean is **0**, and rescale it so that the standard deviation is **1**. Standardizing does not change the **shape** of the distribution.

The Normal model:

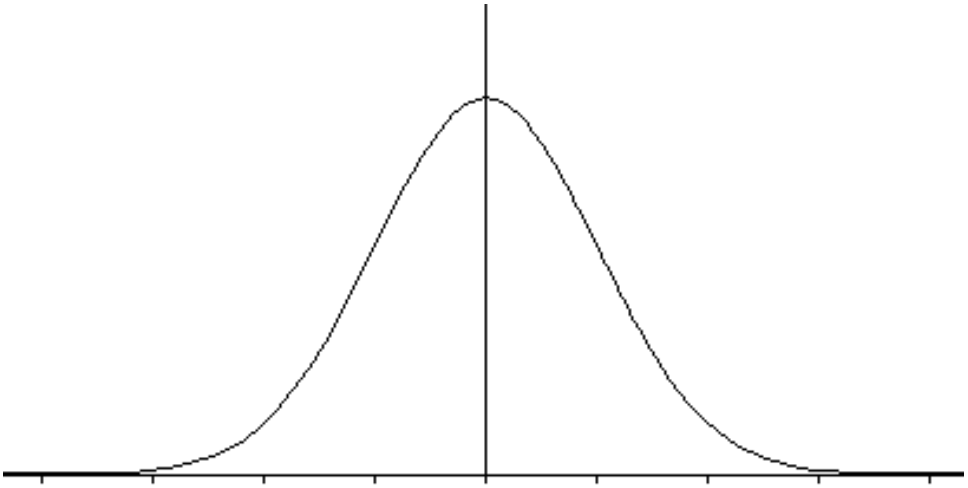
- is **symmetric** and **bell-shaped**.
- follows the **68-95-99.7 Rule**
  - About **68%** of the values fall within **one** standard deviation of the mean.
  - About **95%** of the values fall within **two** standard deviations of the mean.
  - About **99.7%** (almost all) of the values fall within **three** standard deviations of the mean.



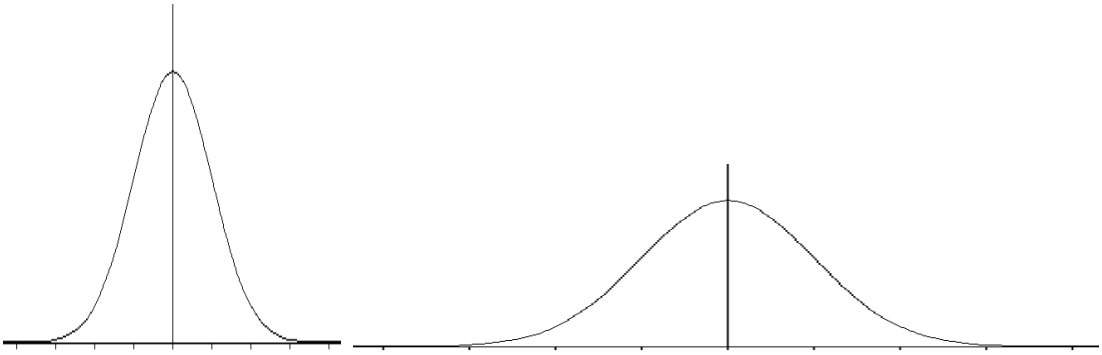
The standard Normal model has mean **0** and standard deviation **1**.



The Normal model is determined by **sigma** and **mu**. We use the Greek letters sigma and mu because this is a **model**; it does not come from actual **data**. Sigma and mu are the **parameters** that specify the model.



The larger sigma, the **more** spread out the normal model appears. The inflection points occur a distance of **sigma** on either side of **mu**.



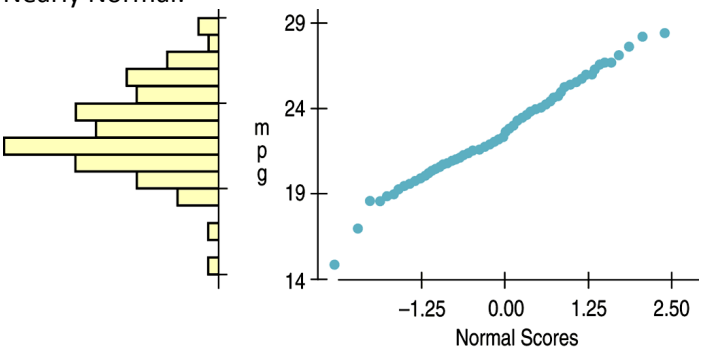
To standardize Normal data, subtract the **mean** (**mu**) and divide by the **standard deviation** (**sigma**).

$$z = \frac{y - \mu}{\sigma}$$

To assess normality:

- Examine the **shape** of the histogram or stem-and-leaf plot. A normal model is **symmetric** about the mean and **bell-shaped**.
- Compare the mean and median. In a Normal model, the mean and median are **equal**.
- Verify that the **68-95-99.7 Rule** holds.
- Construct a **normal probability plot**. If the graph is linear, the model is Normal.

Nearly Normal:



Skewed distribution: