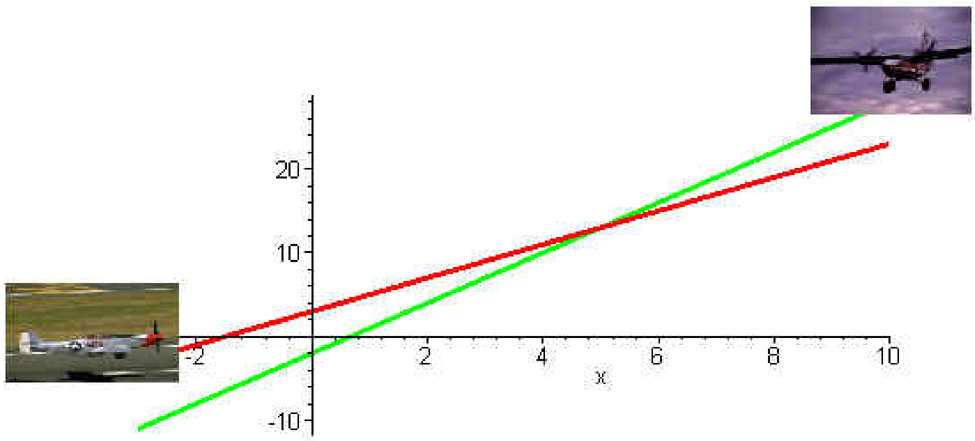
9.2 Plane Curves and Parametric Equations

The following graph shows the position (x, y) of an airplane, where x represents the horizontal distance and y represents the vertical distance. Suppose one airplane moves along the path $y = 2x + 3$ and another moves along the path $y = 3x - 2$. Will the airplanes collide?



Graph Simultaneously:

$x_1 = t$

$y_1 = 2t + 3$

$x_2 = 2t$

$y_2 = 6t - 2$

$T : [-2, 10]_{0.1}$

$X : [-2, 10]_1$

$Y : [-10, 20]_2$

Although the lines intersect, we cannot conclude that the planes will collide, because the graph shows ___________ the planes will be, but not ___________. To figure out when the planes will be at a particular point, we need to introduce a third variable, _____, called a _________________.

Definition of a Plane Curve

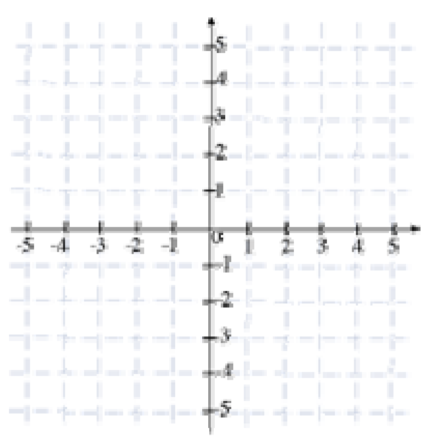
If f and g are continuous functions of t on an interval I, then the equations $x = f(t)$ and $y = g(t)$ are called _________________ equations and _____ is called the _________________. The set of points (x, y) is called the ___________ of the parametric equations. Together, the parametric equations and the graph are called a ___________ ___________, denoted by _____. Plotting points in order of increasing values of t gives a curve with a specific direction, called the _________________ of the curve.

Example 1: Sketching a Curve

Sketch the curve given by the parametric equations:

$x = t^2 - 4$ and $y = \frac{t}{2}$, $-2 \le t \le 3$.

t	-2	-1	0	1	2	3
x						
y						



*Note: Different sets of parametric equations can have the ___________ graph.

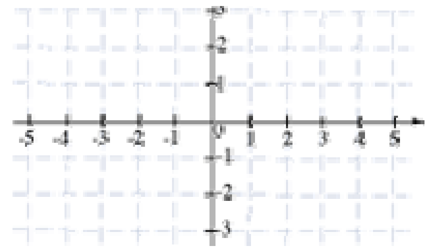
For example, verify $x = 4t^2 - 4$ and $y = t$, $-1 \le t \le \frac{3}{2}$ have the same graph but a greater speed.

Eliminating the Parameter

To eliminate the parameter, solve one of the parametric equations for _____ and substitute into the other equation.

Convert the ____________ equations $x = t^2 - 4$ and $y = \frac{t}{2}$ into _____________ form.

Since $y = \frac{t}{2}$, we know $t = \boxed{}$, and $x = (\quad)^2 - 4$.

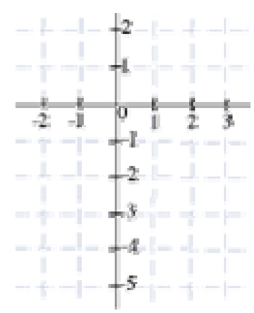


$\boxed{x + 4 = }$ represents a _________________ with a _________________ axis and vertex at _________.

Example 2: Adjusting the Domain After Eliminating the Parameter

Sketch the curve represented by $x = \frac{1}{\sqrt{t+1}}$ and $y = \frac{t}{t+1}$, $t > -1$ by eliminating the parameter and adjusting the domain of the resulting rectangular equation.

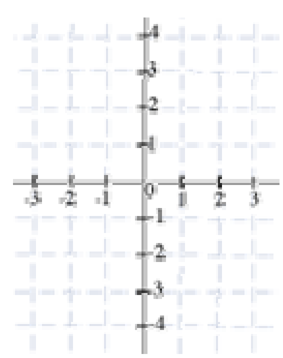
t	-1	0	1	2	3
x					
y					



Example 3: Using Trigonometry to Eliminate a Parameter

Sketch the curve represented by $x = 3\cos\theta$ and $y = 4\sin\theta$, $0 \le \theta \le 2\pi$ by eliminating the parameter finding the corresponding rectangular equation.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x					
y					



Example 4a: Finding Parametric Equations for a Given Graph

Find a set of parametric equations to represent the graph of $y = 1 - x^2$.

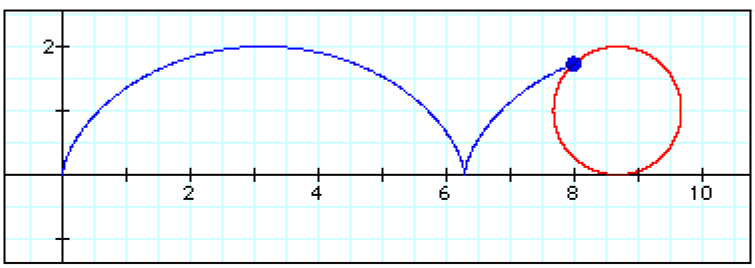
Example 4b: Finding Parametric Equations for a Given Graph

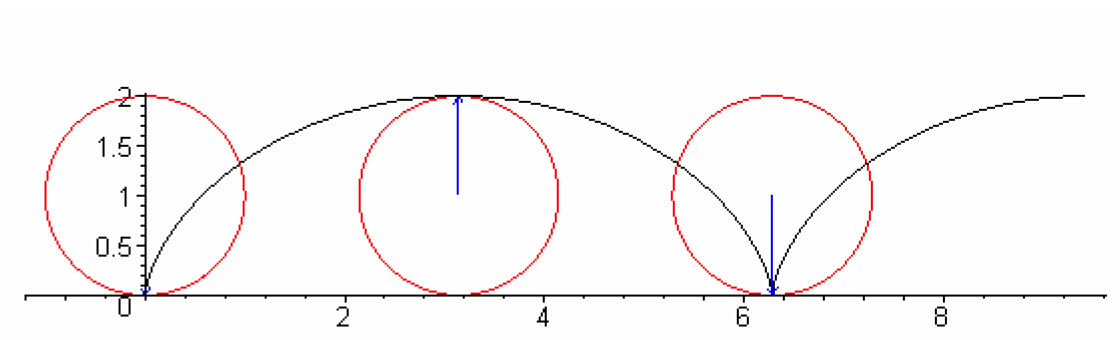
Find a set of parametric equations to represent the graph of $y = 1 - x^2$ if the slope $m = \frac{dy}{dx}$ at the point (x, y).

$m = \frac{dy}{dx} =$

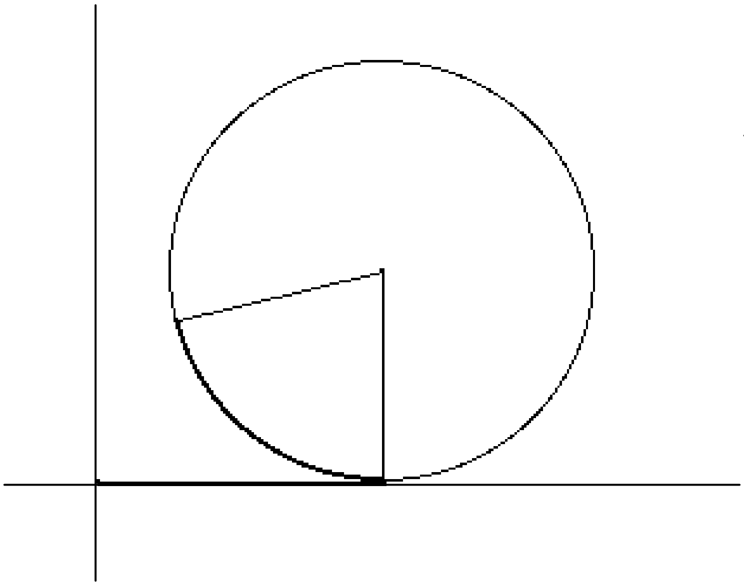
Example 5: Parametric Equations for a Cycloid

Determine the curve traced by a point P on the circumference of a circle of radius r rolling along a straight line in a plane. Such a curve is called a _________________.





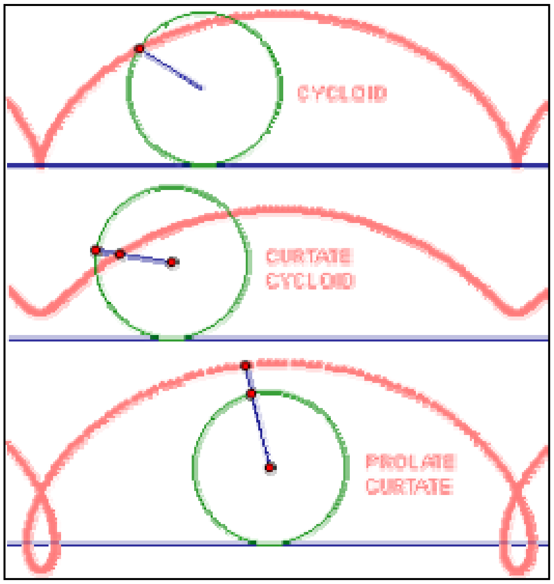
Let the distance rolled from the origin be $|OT| = arcPT = r\theta$.



So the center is:

$x = |OT| - \qquad = \qquad\qquad =$

$y = |TC| - \qquad = \qquad\qquad =$



The equation of the cycloid is given by the parametric equations:

$x(\theta) =$

$y(\theta) =$

and

$x'(\theta) = \qquad\qquad = 0 \quad when \quad \theta =$

$y'(\theta) = \qquad\qquad = 0 \quad when \quad \theta =$

Definition of a Smooth Curve

A curve represented by $x = f(t)$ and $y = g(t)$ on an interval I is called a ___________ ___________ if f' and g' are continuous on I and not simultaneously ___________, except possibly at the endpoints of I.

The curve C is called ________________ ___________ if it is smooth on each subinterval of some partition of I.

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#3, 6, 7, 11, 14, 15, 17, 18, 21, 22, 26, 29, 37, 38, 43, 46, 52, 53, 55, 57, 58, 59